On Throughput of MIMO-OFDM Systems with Joint Iterative Channel Estimation and Multiuser Detection under Different Multiple Access Schemes

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***Abstract*—In this letter we compare the throughput performance of multiuser MIMO-OFDM systems operating in time-variant scenarios. The system employs packet-based transmissions and the receiver is twofold-iterative implementing multiuser detection, channel estimation and soft single-user decoding. Different combinations in terms of multiple-access techniques (time division, frequency division, and space division) and modulation schemes (BPSK and QPSK) are compared. Large-size constellations with interfering users in space-division multiple access (SDMA) are preferable in high-SNR range.**

***Index Terms*—Channel estimation, iterative receivers, MIMO-OFDM systems, multiuser detection, throughput performance.**

I. Introduction

SYSTEM DESIGN for wireless broadband communications mainly aims at providing high data rates with high quality of service, supporting high mobility, working in interference-limited scenarios. Currently, two fundamental technologies are very popular [1]: Multiple-Input Multiple-Output (MIMO) systems, obtained via multiple antennas at both transmitter and receiver locations, capable to increase channel capacity and/or reliability via multiplexing and/or diversity; Orthogonal Frequency Division Multiplexing (OFDM) modulation, capable to simplify significantly channel equalization at the receiver and increase spectral efficiency.

MIMO-OFDM systems [2] represent a *de facto* standard for next-generation wireless systems. Moving from the concept of turbo equalization [3], [4], advanced receivers for MIMO-OFDM systems were recently proposed [5], combining iteratively channel estimation with multiuser detection and soft single-user decoding, and tested with real-world measurements [6]. The potential benefit of multiuser interference in MIMO systems was investigated from an information-theory point of view [8]. Advanced processing for channel estimation within the iterative loop based on Slepian-basis expansion was first proposed in the context of multi-carrier code-division multiple access [7]. Recent works proposed the combination of iterative receivers with joint multiuser detection, channel estimation, and capacity-achieving codes (LDPC codes in [9] and turbo-codes in [10]). The twofold-iterative structure provides beneficial effects in terms of Bit Error Rate (BER) and complexity. Their effectiveness was tested at the physical layer.

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Analogously to [11], we compare the throughput performance of different multiuser MIMO-OFDM systems with twofold-iterative receiver [10] in a time-variant scenario . The objective is to extend the performance test of this recent system architecture to the MAC layer. We assume that users may be separated along time, frequency or space dimension (i.e. orthogonal in the first and second cases, interfering in the third case) and transmit BPSK or QPSK symbols.

Notation - Column vectors (resp. matrices) are denoted with lower-case (resp. upper-case) bold letters, a_n (resp. $A_{n,m}$) being the nth entry of $\boldsymbol{a}$ (resp. the (n,m)th entry of $\boldsymbol{A}$); $\mathrm{diag}(\boldsymbol{a})$ denotes a diagonal matrix whose main diagonal is $\boldsymbol{a}$; $\boldsymbol{I}_N$ is the $N \times N$ identity matrix; $\boldsymbol{i}_N^{(n)}$ is the nth column of $\boldsymbol{I}_N$; $\mathbb{E}\{\cdot\}$, $(\cdot)^{\mathrm{T}}$ and $(\cdot)^{\mathrm{H}}$ denote expectation, transpose and conjugate transpose operators; $\otimes$ denotes the Kronecker matrix product; $\sim \mathcal{N}_{\mathbb{C}}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means distributed as a circular symmetric complex normal variable with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$.

II. System Model

We consider a multiuser MIMO-OFDM system in uplink communications with U users, each with K_u transmit antennas, and one access point with N receive antennas. The whole system has $K = UK_u$ transmit antennas. We assume that each transmit antenna sends an independent data stream encoded by a rate $1/2$ turbo code built upon convolutional encoders with generators $(7,5)_8$. Codewords span both time and frequency dimensions and transmission is frame-oriented. A frame is composed of S OFDM blocks (each with M subcarriers): S_p pilot OFDM blocks for channel estimation plus one single codeword in the remaining $S - S_p$ OFDM blocks. Pilot symbols are placed according to regular patterns as in [7], [10]. Each frame of MS symbols conveys $L = M(S - S_p)$ coded bits (resp. $L = 2M(S-S_p)$) and $L_b = M(S-S_p)/2-2$ information bits (resp. $L_b = M(S - S_p) - 2$) in the BPSK case (resp. in the QPSK case). Notice that two tail bits are used in the turbo code to enforce the final state.

We denote $b_k[\ell]$ and $c_k[\ell]$ the ℓth source bit and the ℓth code bit (including pilots) transmitted by the kth transmit antenna, while referring to the kth transmit antenna, the nth receive antenna, the mth subcarrier, and the sth OFDM block: $x_k[m,s]$ is the transmitted symbol, $H_{n,k}[m,s]$ is the channel coefficient, $w_n[m,s]$ is the additive noise, and $r_n[m,s]$ is the received signal. Also, we denote $\boldsymbol{x}[m,s] = (x_1[m,s], \ldots, x_K[m,s])^{\mathrm{T}}$ the transmitted vector, $\boldsymbol{r}[m,s] = (r_1[m,s], \ldots, r_N[m,s])^{\mathrm{T}}$ the received vector, $\boldsymbol{h}_{(k)}[m,s] = (H_{1,k}[m,s], \ldots, H_{N,k}[m,s])^{\mathrm{T}}$ the kth channel vector, $\boldsymbol{H}[m,s] = \left(\boldsymbol{h}_{(1)}[m,s], \ldots, \boldsymbol{h}_{(K)}[m,s]\right)$ the channel matrix, and $\boldsymbol{w}[m,s] = (w_1[m,s], \ldots, w_N[m,s])^{\mathrm{T}} \sim$

$\mathcal{N}_{\mathbb{C}}(\mathbf{0}, \sigma_w^2 \boldsymbol{I}_N)$ the noise vector. Assuming that both the channel delay spread and the asynchronism among users does not exceed the length of the cyclic prefix, the discrete-time model for the received signal is

$$\boldsymbol{r}[m,s] = \boldsymbol{H}[m,s]\boldsymbol{x}[m,s] + \boldsymbol{w}[m,s] \ . \tag{1}$$

Denoting $\tilde{\boldsymbol{x}} = (\tilde{x}_1, \ldots, \tilde{x}_K)^{\mathrm{T}}$ with $\tilde{x}_k$ representing the soft-information on symbol x_k from the single-user decoders, multiuser detection is based on parallel interference cancellation

$$\tilde{\boldsymbol{r}}_{(k)} = \boldsymbol{r} - \boldsymbol{H}\tilde{\boldsymbol{x}}_{(k)} \ , \tag{2}$$

where $\tilde{\boldsymbol{x}}_{(k)} = \tilde{\boldsymbol{x}} - \tilde{x}_k \boldsymbol{i}_K^{(k)}$ is the residual term from the interference cancellation, and unbiased MMSE filtering

$$\tilde{z}_k = \frac{\boldsymbol{i}_K^{(k)\mathrm{H}} \left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \sigma_w^2 \boldsymbol{V}_{(k)}^{-1}\right)^{-1} \boldsymbol{H}^{\mathrm{H}}\tilde{\boldsymbol{r}}_{(k)}}{\boldsymbol{i}_K^{(k)\mathrm{H}} \left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \sigma_w^2 \boldsymbol{V}_{(k)}^{-1}\right)^{-1} \boldsymbol{H}^{\mathrm{H}}\boldsymbol{h}_{(k)}} \ , \tag{3}$$

where $\boldsymbol{V}_{(k)} = \mathrm{diag}(1 - |\tilde{x}_1|^2, \ldots, 1 - |\tilde{x}_{k-1}|^2, 1, 1 - |\tilde{x}_{k+1}|^2, \ldots, 1 - |\tilde{x}_K|^2)$ is the variance vector.

Each data stream $\{\tilde{z}_k[1], \ldots, \tilde{z}_k[L]\}$ is independently decoded with a turbo-decoding algorithm (based on log-domain BCJR algorithm [12]) using an equivalent channel model with zero-mean additive gaussian noise whose variance is $\eta_k^2 = \frac{1}{\boldsymbol{i}_K^{(k)\mathrm{H}} \left(\boldsymbol{H}^{\mathrm{H}}\boldsymbol{H} + \sigma_w^2 \boldsymbol{V}_{(k)}^{-1}\right)^{-1} \boldsymbol{H}^{\mathrm{H}}\boldsymbol{h}_{(k)}}$. It is worth noticing that $\tilde{z}_k[\ell]$ has been transmitted on the mth subcarrier during the sth OFDM block if $\ell = (s-1)M + m$.

Channel estimation is based on the Slepian expansion $H_{n,k}[m,s] \approx \sum_{i=1}^{I} \psi_{n,k}[m,i] u_i[s]$, where: the maximum normalized Doppler spread ($\nu_{\max}$) is assumed to be known; $\psi_{n,k}[m,i]$ is the ith Slepian coefficient for the mth subcarrier on the link between the kth transmit antenna and the nth receive antenna; $u_i[s]$ is the sth sample of the ith Slepian sequence associated to the time interval $s = 1, \ldots, S$ with frequency support $[-\nu_{\max}, \nu_{\max}]$ and corresponding eigenvalue λ_i; the approximate signal space extension is $2\nu_{\max}S + 1 < I \leq S$, see [13] for more details. Denoting $\boldsymbol{u}[s] = (u_1[s], \ldots, u_I[s])^{\mathrm{T}}$ and $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_I)^{\mathrm{T}}$, and (referring to the mth subcarrier) $\boldsymbol{r}[m] = \left(\boldsymbol{r}^{\mathrm{T}}[m,1] \ldots, \boldsymbol{r}^{\mathrm{T}}[m,S]\right)^{\mathrm{T}}$, $\boldsymbol{\Xi}[m,s] = \boldsymbol{I}_N \otimes (\boldsymbol{x}[m,s] \otimes \boldsymbol{u}[s])^{\mathrm{T}}$, $\boldsymbol{\Xi}[m] = \left(\boldsymbol{\Xi}^{\mathrm{T}}[m,1], \ldots, \boldsymbol{\Xi}^{\mathrm{T}}[m,S]\right)^{\mathrm{T}}$, $\boldsymbol{\psi}_{n,k}[m] = (\psi_{n,k}[m,1], \ldots, \psi_{n,k}[m,I])^{\mathrm{T}}$, $\boldsymbol{\psi}_n[m] = \left(\boldsymbol{\psi}_{n,1}^{\mathrm{T}}[m,1], \ldots, \boldsymbol{\psi}_{n,K}^{\mathrm{T}}[m,S]\right)^{\mathrm{T}}$, $\boldsymbol{\psi}[m] = \left(\boldsymbol{\psi}_1^{\mathrm{T}}[m], \ldots, \boldsymbol{\psi}_N^{\mathrm{T}}[m]\right)^{\mathrm{T}}$, $\boldsymbol{w}[m] = \left(\boldsymbol{w}^{\mathrm{T}}[m,1], \ldots, \boldsymbol{w}^{\mathrm{T}}[m,S]\right)^{\mathrm{T}}$, the signal model for channel estimation is

$$\boldsymbol{r}[m] = \boldsymbol{\Xi}[m]\boldsymbol{\psi}[m] + \boldsymbol{w}[m] \ . \tag{4}$$

Omitting the subcarrier index m for sake of simplicity, the linear MMSE estimate is

$$\hat{\boldsymbol{\psi}} = \left(\hat{\boldsymbol{\Xi}}^{\mathrm{H}}\boldsymbol{\Delta}^{-1}\hat{\boldsymbol{\Xi}} + \boldsymbol{C}_{\psi}^{-1}\right)^{-1} \hat{\boldsymbol{\Xi}}^{\mathrm{H}}\boldsymbol{\Delta}^{-1}\boldsymbol{r} \ , \tag{5}$$

where $\boldsymbol{C}_{\psi} = \frac{1}{2\nu_{\max}} \boldsymbol{I}_{NK} \otimes \mathrm{diag}(\boldsymbol{\lambda})$ denotes the covariance matrix of the Slepian coefficients; $\hat{\boldsymbol{\Xi}}$ contains the expected transmitted symbols computed via *a posteriori* information from SISO decoders; and where $\boldsymbol{\Delta} = \boldsymbol{\Theta} + \sigma_w^2 \boldsymbol{I}_{NS}$, $\boldsymbol{\Theta} = \mathrm{diag}(\boldsymbol{\vartheta}) \otimes \boldsymbol{I}_N$, $\boldsymbol{\vartheta} = (\vartheta_1, \ldots, \vartheta_S)^{\mathrm{T}}$, $\vartheta_s = \sum_{k=1}^{K} \left(1 - |\hat{x}_k[m,s]|^2\right)$.

TABLE I

PARAMETERS FOR THE CONSIDERED CONFIGURATIONS: SYSTEMS WITH $U = 2$ USERS (UPPER PART) AND WITH $U = 3$ USERS (LOWER PART).

	M	S	S_p	θ_u (Mbps)	θ_s (Mbps)
BPSK-TDMA	60	100	10	6.75	6.75
BPSK-FDMA	30	200	20	3.37	6.75
BPSK-SDMA	60	100	10	6.75	13.49
QPSK-TDMA	60	50	5	13.49	13.49
QPSK-FDMA	30	100	10	6.75	13.49
QPSK-SDMA	60	50	5	13.49	26.98
BPSK-TDMA	60	100	10	6.75	6.75
BPSK-FDMA	20	300	30	2.25	6.75
BPSK-SDMA	60	100	10	6.75	20.24
QPSK-TDMA	60	50	5	13.49	13.49
QPSK-FDMA	20	150	15	4.50	13.49
QPSK-SDMA	60	50	5	13.49	40.47

III. RESULTS AND DISCUSSION

Computer simulations were performed with Matlab, with uniform *a priori* distribution of the source bits, in order to obtain the user BER (P_e). Time-variant channels were simulated with Rayleigh fading statistics according to the model in [14]. Maximum normalized Doppler was set to $\nu_{\max} = 0.25 \cdot 10^{-3}$, corresponding for a system operating at 2 GHz at a maximum speed of 34 km/h (i.e. vehicles in urban areas). Similar results to those shown were obtained for different maximum Doppler spreads.

The following parameters were selected: $L_b = 2698$ source bits per frame, 10% of pilot bits (placed as in [10]) with respect to the code bits in a frame, $M = 60$ available subcarriers, and OFDM-block duration of $T_s = 4\ \mu s$. Six different configurations were compared combining BPSK and QPSK modulations with Time-Division (TD), Frequency-Division (FD), and Space-Division (SD) multiple access (MA) schemes in systems with $U = 2, 3$ users and $N = 2$ receive antennas. Only the case with $K_u = 1$ transmit antenna per user is shown. The results are easily extended to the case $K_u > 1$ assuming that each user parallelizes its data stream into K_u different data streams [10]. Also, large-size QAM constellations are easily considered with similar data processing.

Different MA schemes have different impact on the performance of the multiuser detector, depending on the number of interfering users. TDMA and FDMA allocate orthogonal resources, respectively in time and frequency domains, to different users. They do not experience interfering users, thus the only source of interference is represented by the multiple antennas at the user location. On the other hand SDMA allocates the same time/frequency resources to each user that represents a source of interference for the remaining ones. The parameters of each configuration are reported in Table I, where

$$\theta_u = \frac{K_u L_b}{S T_s} \quad , \quad \theta_s = \begin{cases} \theta_u & \text{for TDMA} \\ U\theta_u & \text{for FDMA, SDMA} \end{cases} \ ,$$

represent user and system transmission rates, respectively. It is worth noticing that, assuming a fixed total bandwidth, the number of subcarriers assigned to each user in the TDMA and SDMA schemes is M while in the FDMA scheme is M/U.

Referring to a packet-based transmission scheme, with probability of retransmission $P_R = 1 - (1-P_e)^{L_b}$, the average number of retransmissions is $\sum_{n=1}^{+\infty} n P_R^{n-1}(1 - P_R) = \frac{1}{1-P_R}$, thus average user and system throughputs are respectively

$$\eta_u = \theta_u(1-P_e)^{L_b} \quad , \quad \eta_s = \begin{cases} \eta_u & \text{for TDMA} \\ U\eta_u & \text{for FDMA, SDMA} \end{cases} .$$

Figure 1 compares the BER performance with respect to the Signal-to-Noise Ratio (SNR) for the six configurations. Although BER curves characterize physical-layer performance, such a comparison does not take into account the different transmission rates (both for the single user and for the whole system), thus MAC-layer performance in terms of user throughput and system throughput have been considered. Figure 2 compares the corresponding performance in terms of system throughput. Referring to systems with $U = 2$ users, it is apparent how in the low-SNR range ($0-2$ dB) small-size constellations and orthogonal (interference-free) transmissions should be used to guarantee the information flow. In the medium-SNR range ($2-5$ dB) orthogonal transmissions may be used in combination with larger-size constellations, or alternatively small-size constellations may be combined with interfering transmissions (i.e. SDMA). Interference increases with SNR, however multiuser detection in SDMA schemes is able to exploit the multiuser diversity, turning interference from a limiting-performance factor to a beneficial issue. In the high-SNR range (above 5 dB) the interference dominates over the additive noise and multiuser detection is able to benefit enormously from it. The figures show better performance both for user and system throughput, with system throughput achieving the maximum transmission rate, thus larger-size constellation and interfering transmissions are encouraged. Referring to systems with $U = 3$ users, we have the same behavior with larger SNR thresholds. Such behavior is representative of both underloaded ($K \leq N$) and overloaded ($K > N$) systems. It is worth noticing that in underloaded systems the threshold for high-SNR range is very low (almost 5 dB), as the receiver employs advanced processing combining multiuser-detection, turbo-equalization, and turbo-decoding.

As for the computational complexity at the receiver, the MA scheme has impact on the size of the discrete-time model in Eq. (1), thus affecting the complexity of both multiuser detection and channel estimation. The complexity is mainly due to the matrix inversions in Eqs. (3) and (5). Assuming that the complexity for inverting a square matrix of size N is $\mathcal{O}(N^3)$, the required complexity is $\mathcal{O}((UK_u)^3)$ and $\mathcal{O}((NIUK_u)^3)$, respectively, in the SDMA case, while $\mathcal{O}(U(K_u)^3)$ and $\mathcal{O}(U(NIK_u)^3)$, respectively, in both TDMA and FDMA cases. It is apparent that SDMA scheme requires U^2 times the complexity of TDMA or FDMA schemes.

Summarizing, in high-SNR range, SDMA with large-size constellation may be preferred in terms of system throughput despite the presence of major interference, even in overloaded scenarios, at price of higher complexity.

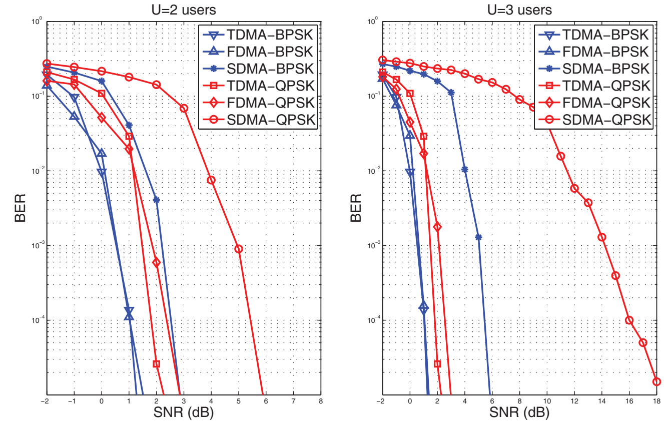

Fig. 1. Performance in terms of BER vs SNR.

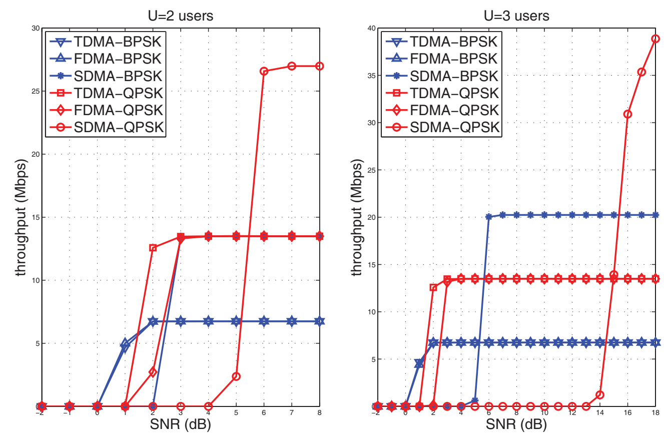

Fig. 2. Performance in terms of system throughput vs SNR.

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